Choice Under Risk

Contents

Question 1: Risk vs. Ambiguity

Guiding Question

Q1. What is the difference between risk and ambiguity? Give a few examples of decisions made under risk and decisions made under ambiguity, respectively.

Risk refers to situations where the possible outcomes and their associated probabilities are **known**. The decision-maker faces uncertainty, but that uncertainty is quantifiable.

Ambiguity (also called **Knightian uncertainty**) refers to situations where the probabilities of outcomes are **unknown or unquantifiable**. The decision-maker does not even know the likelihood of different outcomes.

	Risk	**Ambiguity**
Outcomes known?	Yes	Yes
Probabilities known?	Yes	No

Examples

Decisions under Risk:

- Flipping a fair coin and betting on heads (probability = 50% is known).
- Buying a lottery ticket with a published odds structure.
- Rolling a standard die and betting on a specific number (probability = $\frac{1}{6}$).
- An insurance company pricing a policy based on actuarial statistics.

Decisions under Ambiguity:

- Investing in a brand-new, untested market where no historical data exists.
- A company deciding whether to enter an unknown foreign market.
- Choosing whether to carry an umbrella when visiting a city you have never been to, with no weather forecast available.
- Early-stage drug development, where the probability of clinical success is genuinely unknown.

Question 2: Risk Attitudes

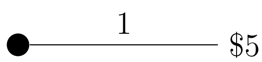
Guiding Question

Q2. Consider the two options below:
- Option A: Receive \$5 for sure.
- Option B: A 50% chance to win \$10, and a 50% chance to win \$0.

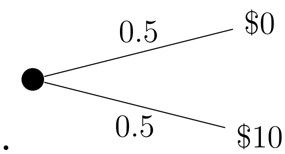
Abby strictly prefers Option A to Option B, Neil is indifferent between Option A and Option B, and Sally strictly prefers Option B to Option A. What does this say about each of their risk attitudes?

Question 2:

Option A:



Option B:



The expected value (EV) of the two options is:

$$EV(A) = \$5$$

$$EV(B) = 0.5 \times \$10 + 0.5 \times \$0 = \$5$$

Both options have the **same expected value of \$5**. Yet the three individuals choose differently:

- **Abby** strictly prefers the certain \$5 (Option A) over the gamble (Option B). She is willing to give up the chance of winning more in order to avoid uncertainty. This reveals that Abby is **risk averse**.
- **Neil** is indifferent between Option A and Option B. Since both options have the same expected value and he does not favour one over the other, Neil is **risk neutral**. He cares only about expected value, not the variability around it.
- **Sally** strictly prefers the gamble (Option B) over the certain \$5 (Option A). She is attracted to the possibility of winning more, even though the expected value is the same. This reveals that Sally is **risk seeking** (or risk loving).

Question 3: The St. Petersburg Paradox

Guiding Question

Q3. What is the St. Petersburg Paradox? Why does it challenge Expected Value (EV) as a useful decision rule?

The Setup

The St. Petersburg game is defined as follows:

- A fair coin is flipped repeatedly until it lands on **tails**.
- If tails appears on flip n, the player wins 2^n dollars.

The **expected value** of this game is:

$$EV = \sum_{n=1}^{\infty} \frac{1}{2^n} \cdot 2^n = \sum_{n=1}^{\infty} 1 = \infty$$

The Paradox

The game has an **infinite expected value**, yet virtually no one would pay more than a small amount (say, \$20–\$30) to play it. If people were rational EV-maximisers, they should be willing to pay any finite amount to play — but they clearly do not.

Why It Challenges EV as a Decision Rule

The paradox demonstrates that **Expected Value alone is not a psychologically realistic or descriptively accurate guide to decision-making** under risk. People do not treat a tiny probability of an astronomically large payoff as equivalent to a moderate certain gain, even if the mathematics says the expected values are equal or infinite.

This motivated Daniel Bernoulli's insight that people maximise **expected utility** — the expected value of a *concave* utility function of wealth — rather than expected monetary value. Under a concave utility function (e.g., $u(x) = \ln(x)$), the expected utility of the St. Petersburg game is finite, which resolves the paradox.

Question 4: Utility Functions, Certainty Equivalents, and Risk Premiums

Guiding Question

Q4.

1. Draw the utility functions of Abby, Neil, and Sally as described in Question 2.
2. Mark the expected utility of Option A and that of Option B in each figure, and explain how each utility function captures their respective risk attitudes (i.e., how it explains their preferences between the two options).
3. Mark the certainty equivalent of Option B on each figure, and note how it compares to the expected value of Option B.
4. Mark the risk premium in each figure, and explain who — Abby, Neil, or Sally — would be most likely among the three to invest in the stock market.

(1) Utility Functions

The shape of each person's utility function over wealth x reflects their risk attitude:

- **Abby (risk averse):** A **concave** utility function — the curve bends downward, exhibiting diminishing marginal utility of wealth. Example: $u(x) = \sqrt{x}$.
- **Neil (risk neutral):** A **linear** utility function — a straight line with constant marginal utility of wealth. Example: $u(x) = x$.
- **Sally (risk seeking):** A **convex** utility function — the curve bends upward, exhibiting increasing marginal utility of wealth. Example: $u(x) = x^2$.

(2) Expected Utility of Options A and B

For each person, mark the following on the utility function graph:

- $EU(A) = u(\$5)$ — the utility of receiving \$5 with certainty.
- $EU(B) = 0.5 \cdot u(\$10) + 0.5 \cdot u(\$0)$ — a probability-weighted average of utilities.

Graphically, $EU(B)$ is found by taking the **midpoint of the chord** connecting $u(\$0)$ and $u(\$10)$ on the utility curve.

How each function captures risk attitude:

- **Abby** (concave curve): the midpoint of the chord lies *below* the curve, so $EU(B) < u(\$5) = EU(A)$. She prefers A.
- **Neil** (linear curve): the midpoint lies *exactly on* the line, so $EU(B) = u(\$5) = EU(A)$. He is indifferent.
- **Sally** (convex curve): the midpoint lies *above* the curve, so $EU(B) > u(\$5) = EU(A)$. She prefers B.

(3) Certainty Equivalent

The **certainty equivalent** (CE) of Option B is the guaranteed dollar amount that makes the individual indifferent between receiving that amount for certain and taking the gamble. It is found by solving:

$$u(CE) = EU(B)$$

Graphically, CE is the wealth level on the horizontal axis corresponding to $EU(B)$ on the utility curve.

- **Abby (risk averse):** $CE < EV(B) = \$5$. She would accept less than \$5 to avoid the gamble.
- **Neil (risk neutral):** $CE = EV(B) = \$5$. The certain amount equals the expected value.

- **Sally (risk seeking):** $CE > EV(B) = \$5$. She would only give up the gamble for more than its expected value.

(4) Risk Premium and Stock Market Investment

The **risk premium** is the difference between the expected value of the gamble and the certainty equivalent:

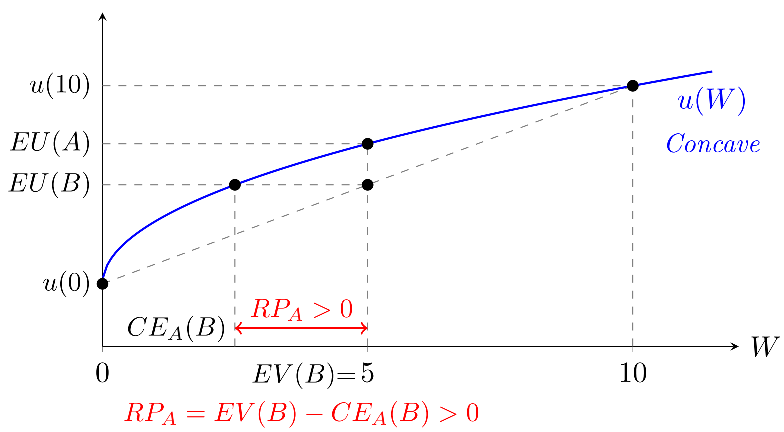
$$RP = EV(B) - CE(B)$$

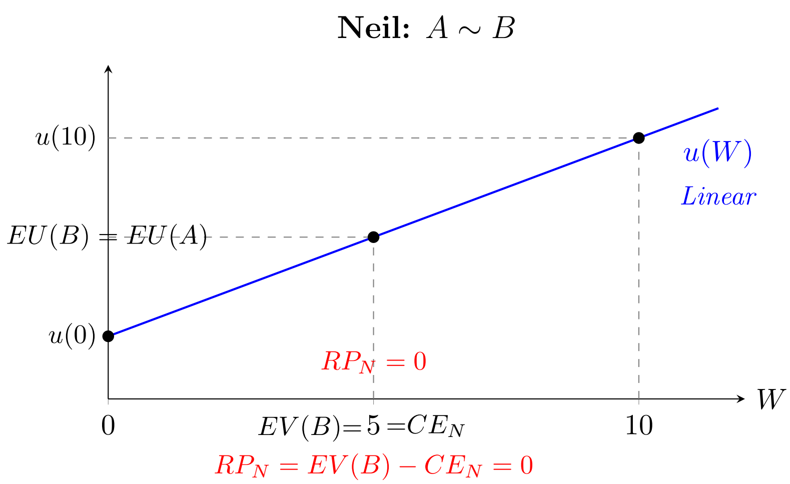
- **Abby:** $RP > 0$ — she is willing to pay a premium to avoid risk.
- **Neil:** $RP = 0$ — no premium; he is indifferent.
- **Sally:** $RP < 0$ — she requires a premium to *give up* the gamble.

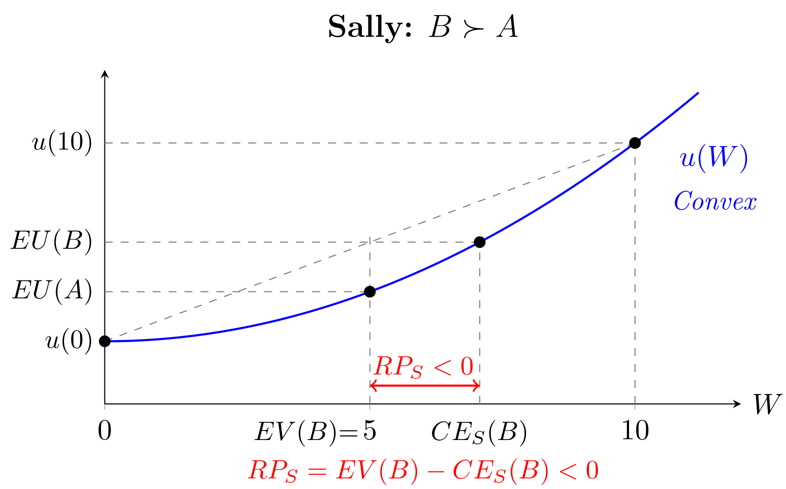
Who is most likely to invest in the stock market?

Sally, the risk-seeking individual, is most likely to invest in the stock market. Stock investing involves accepting variance in returns in exchange for a higher expected payoff — precisely the trade-off that a risk-seeking person finds attractive. Abby, being risk averse, would demand a significant risk premium before investing and might prefer safer assets such as bonds or savings accounts.

Abby: $A \succ B$







Question 5: The Allais Paradox

Guiding Question

Q5. The Allais Paradox

Scenario 1: Choose between:

- A: \$2,500 with probability 0.33; \$2,400 with probability 0.66; \$0 with probability 0.01
- B: \$2,400 with certainty

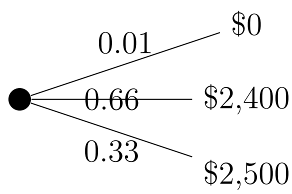
Scenario 2: Choose between:

- C: \$2,500 with probability 0.33; \$0 with probability 0.67
- D: \$2,400 with probability 0.34; \$0 with probability 0.66

Research shows that most people choose B over A, but C over D. Show that this pattern of choices is inconsistent with Expected Utility Theory.

Question 5 - Scenario 1:

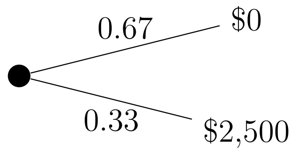
Option A:



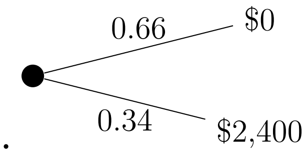
Option B:

Question 5 - Scenario 2:

Option C:



Option D:



Typical Choices

Most people choose **B over A** and **C over D**.

Proof of Inconsistency with Expected Utility Theory

Under EUT, there exists a utility function $u(\cdot)$ such that choices are determined by expected utility. Let $u(0) = 0$ for simplicity.

From the choice of B over A (i.e. $EU(B) \geq EU(A)$):

$$u(2400) \;\geq\; 0.33 \cdot u(2500) + 0.66 \cdot u(2400) + 0.01 \cdot u(0)$$

$$0.34 \cdot u(2400) \;\geq\; 0.33 \cdot u(2500) \tag{1}$$

From the choice of C over D (i.e. $EU(C) \geq EU(D)$):

$$0.33 \cdot u(2500) \;\geq\; 0.34 \cdot u(2400) \tag{2}$$

Inequalities (1) and (2) **directly contradict each other**. Therefore, the majority preference of B over A and C over D **cannot be simultaneously consistent** with any expected utility function. This is the Allais Paradox. ■

What This Tells Us

The paradox shows that people **overweight certainty** relative to near-certain outcomes (the “certainty effect”), violating the *independence axiom* of EUT.

Question 6: The Framing Effect

Guiding Question

Q6. The Framing Effect

Problem 1:
Imagine that the U.S. is preparing for the outbreak of an unusual disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the programs are as follows:

- If Program A is adopted, 200 people will be saved.
- If Program B is adopted, there is a 1/3 probability that 600 people will be saved, and a 2/3 probability that no one will be saved.

Which of the two programs would you favor?

Problem 2:
Imagine the same situation as above. Two alternative programs have been proposed:

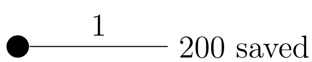
- If Program C is adopted, 400 people will die.
- If Program D is adopted, there is a 1/3 probability that nobody will die, and a 2/3 probability that 600 people will die.

Which of the two programs would you favor?

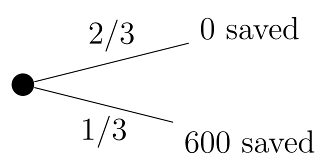
What does the typical pattern of responses across Problems 1 and 2 reveal about how people make decisions? Can this be explained by Expected Utility Theory?

Question 6 - Problem 1:

Program A:



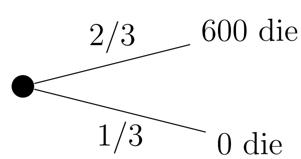
Program B:



Question 6 - Problem 2:

Program C:

Program D:



Typical Responses

- **Problem 1:** Most people choose **Program A** (the certain option — 200 lives saved).
- **Problem 2:** Most people choose **Program D** (the gamble — $\frac{1}{3}$ probability that nobody dies).

The Key Observation

Notice that **Programs A and C are objectively identical**: saving 200 out of 600 is the same as letting 400 die. Similarly, **Programs B and D are identical**. The only difference is how outcomes are framed — in terms of **lives saved** (a gain frame) versus **lives lost** (a loss frame).

Explanation

This pattern reflects the **Framing Effect**: people's choices are influenced by how options are presented, not merely by their objective outcomes.

- In the **gain frame** (Problem 1), people are **risk averse** — they prefer the certain gain of 200 lives saved.
- In the **loss frame** (Problem 2), people are **risk seeking** — they prefer to gamble rather than accept a certain loss of 400 lives.

Can EUT Explain This?

No. Under EUT, a rational agent evaluates outcomes based on their actual consequences, not their description. Since $A \equiv C$ and $B \equiv D$ in terms of outcomes, a consistent EUT maximiser should make the same choice in both problems. The systematic reversal of preferences violates **description invariance**, a basic requirement of rational choice theory.

Question 7: The Reflection Effect

Guiding Question

Q7. The Reflection Effect

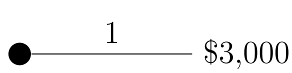
Scenario 1: Choose between:
- A: $3,000 for sure
- B: 80% chance to win $4,000; 20% chance of winning nothing

Scenario 2: Choose between:
- C: Lose $3,000 for sure
- D: 80% chance to lose $4,000; 20% chance of losing nothing

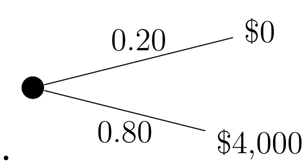
What do the typical choices across these two scenarios reveal? How does this relate to the concept of the reflection effect?

Question 7 - Scenario 1:

Option A:

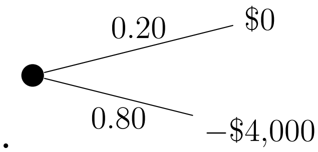


Option B:



Question 7 - Scenario 2:

Option C: 

Option D: 

Typical Choices

- **Scenario 1 (gains):** Most people choose **A** ($3,000 for sure) over B — **risk averse** in the domain of gains.
- **Scenario 2 (losses):** Most people choose **D** (the gamble) over C — **risk seeking** in the domain of losses.

Note the expected values:

$$\begin{aligned} EV(A) &= \$3{,}000 \\ EV(B) &= 0.8 \times \$4{,}000 + 0.2 \times \$0 = \$3{,}200 \end{aligned}$$

Option B has a higher EV, yet most people choose A.

$$\begin{aligned} EV(C) &= -\$3{,}000 \\ EV(D) &= 0.8 \times (-\$4{,}000) + 0.2 \times \$0 = -\$3{,}200 \end{aligned}$$

Option C has a higher EV, yet most people choose D.

The Reflection Effect

The **Reflection Effect** refers to the phenomenon where risk preferences **reverse** when moving from gains to losses. People tend to be:

- **Risk averse** when facing potential **gains**.
- **Risk seeking** when facing potential **losses**.

This is as if the preference pattern "reflects" across the zero-wealth reference point. It directly contradicts EUT, which predicts consistent risk attitudes across the full wealth spectrum, and is a key motivation for the S-shaped value function in Prospect Theory.

Question 8: Reference Dependence

Guiding Question

Q8. Reference Dependence

Scenario 1: Your current salary is $50,000.

- A) Your salary remains $50,000 for sure.
- B) 50% chance your salary rises to $60,000; 50% chance it falls to $40,000.

Scenario 2: Your current salary is $60,000.

- A) Your salary falls to $50,000 for sure.
- B) 50% chance your salary stays at $60,000; 50% chance it falls to $40,000.

Compare your choices across the two scenarios. What do any differences reveal about the role of reference points in decision-making?

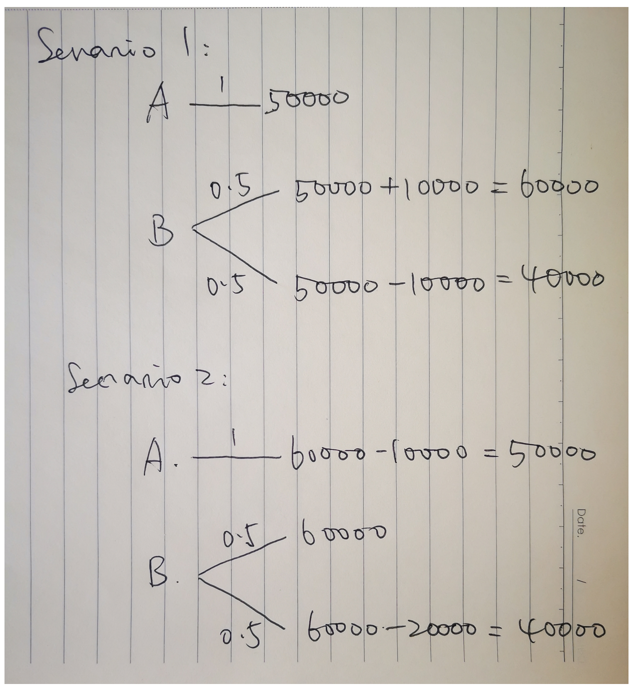


Figure 1: Equivalence of Scenario 1 and Scenario 2

Typical Responses

- **Scenario 1** (reference point = $50,000): Most people choose **A** — keep the salary at $50,000 for sure, showing **risk aversion**.
- **Scenario 2** (reference point = $60,000): Most people choose **B** — take the gamble, showing **risk seeking**.

The Key Observation

The two scenarios offer **objectively identical lotteries** over the same set of final wealth levels ($40,000, $50,000, or $60,000). The only difference is the **reference point** (current salary). Yet people make opposite choices.

Explanation: Reference Dependence

This demonstrates that people do not evaluate outcomes in terms of **final wealth levels** (as EUT assumes), but rather in terms of **gains and losses relative to a reference point** (typically the status quo).

- In **Scenario 1**, the reference is $50,000. Option A is neutral (no change) and Option B involves a possible gain or loss. People are risk averse over gains, so they choose A.
- In **Scenario 2**, the reference is $60,000. Option A is now a **certain loss** of $10,000, and Option B offers a chance to avoid any loss. People are risk seeking over losses, so they switch to B.

This is a foundational insight of **Prospect Theory**: utility is defined over *changes* from a reference point, not absolute wealth levels.

Question 9: The Isolation Effect

Guiding Question

Q9. The Isolation Effect

Problem 1:
Stage 1: 25% chance to move to Stage 2; 75% chance to receive $0.
Stage 2 (if reached): Choose between:

- A) $3,000 for sure
- B) 80% chance to win $4,000; 20% chance of winning nothing

Problem 2: Choose between:

- C) 25% chance to win $3,000
- D) 20% chance to win $4,000

Note that the overall probabilities in Problem 1 (when accounting for Stage 1) are equivalent to those in Problem 2. Do people typically choose consistently across the two problems? What does this reveal?

Comparing the Two Problems

Working out the overall probabilities in **Problem 1**:

$$P(\text{win \$3,000 via A in Stage 2}) = 0.25 \times 1.00 = 0.25 \quad \Longrightarrow \quad \text{equivalent to Option C}$$

$$P(\text{win \$4,000 via B in Stage 2}) = 0.25 \times 0.80 = 0.20 \quad \Longrightarrow \quad \text{equivalent to Option D}$$

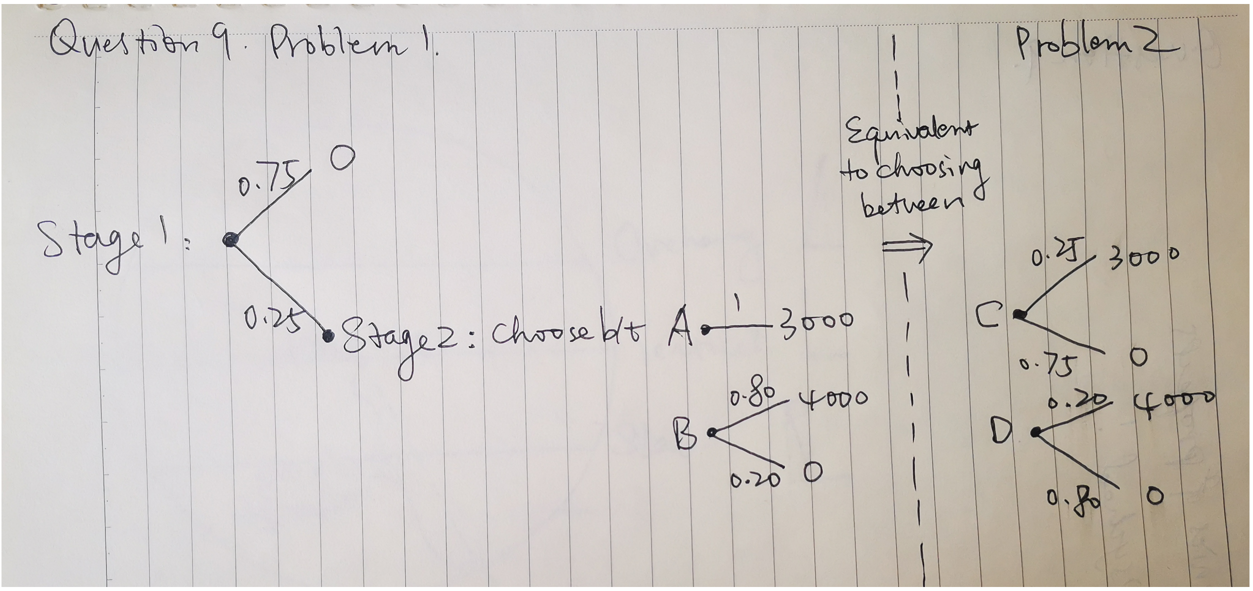


Figure 2: Equivalence of Problem 1 and Problem 2

So **Problem 1** (choosing A or B in Stage 2) is **mathematically equivalent** to **Problem 2** (choosing C or D directly). A consistent EUT agent should make the same choice in both.

Typical Choices

- In **Problem 1**, most people choose **A** in Stage 2 — risk averse, mirroring Scenario 1 in Question 7.
- In **Problem 2**, most people choose **D** (20% chance of $4,000 over 25% chance of $3,000) — risk seeking.

This is an **inconsistency**: people choose A (equivalent to C) in one frame but D over C in the other.

The Isolation Effect

People tend to **ignore** (isolate) the shared or earlier components of a problem and focus only on the stage where the active choice is made. In Problem 1, the Stage 1 lottery is mentally set aside, and the Stage 2 choice is treated as though reaching Stage 2 is certain — triggering risk aversion. In Problem 2, without that framing, people evaluate the raw probabilities differently.

This violates EUT's requirement that choices depend only on **final outcome probabilities**, not on how those probabilities are decomposed or presented.

Question 10: Can EUT Explain Questions 6–9?

Guiding Question

Q10. Can the majority choices observed in Questions 6–9 be explained by Expected Utility Theory? Why or why not?

No. Expected Utility Theory cannot explain the majority choices in Questions 6–9. The table below summarises why:

Question	Phenomenon	Why EUT Fails
Q6: Framing Effect	Same outcomes, different frames ⇒ different choices	EUT requires description invariance; how options are described should be irrelevant.
Q7: Reflection Effect	Risk averse in gains, risk seeking in losses	A globally concave $u(\cdot)$ cannot produce systematic preference reversals around a reference point.
Q8: Reference Dependence	Same final wealth levels ⇒ different choices depending on the reference point	EUT evaluates final wealth only; reference points play no role under EUT.
Q9: Isolation Effect	Mathematically equivalent problems ⇒ different choices depending on presentation	EUT requires consistent evaluation of final probabilities, regardless of how they are decomposed.

Table 1: Summary of behavioural anomalies and why they cannot be explained by Expected Utility Theory.

All four phenomena share a common thread: **people's choices are sensitive to factors that EUT considers irrelevant** — frames, reference points, and the decomposition of probabilities. These systematic violations motivated the development of **Prospect Theory**.

Question 11: Prospect Theory

Guiding Question

Q11.

1. Draw the value function and the probability weighting function of Prospect Theory.
2. List as many characteristics of each function as you can (aim for at least three per function).
3. What do the two functions say about the risk attitudes of decision-makers?
4. Explain the four-fold pattern of risk attitudes using a 2×2 matrix.

(1) & (2) The Value Function

The **value function** $v(x)$ is defined over **gains and losses relative to a reference point** (not final wealth). Its key characteristics are:

1. **Defined over changes:** The argument is $x = \text{outcome} - \text{reference point}$, not final wealth.
2. **S-shaped:** Concave in the domain of gains ($x > 0$); convex in the domain of losses ($x < 0$).
3. **Reference point at the origin:** $v(0) = 0$; the curve passes through the origin.
4. **Loss aversion:** The function is **steeper for losses than for gains** — losing $\$X$ hurts more than gaining $\$X$ feels good. Formally: $|v(-x)| > v(x)$ for $x > 0$. Empirically, the loss-aversion coefficient $\lambda \approx 2.25$.
5. **Diminishing sensitivity:** The marginal impact of outcomes diminishes as you move further from the reference point in either direction.

A commonly used parametric form is:

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x < 0 \end{cases}$$

where empirical estimates give $\alpha, \beta \approx 0.88$ and $\lambda \approx 2.25$.

(1) & (2) The Probability Weighting Function

The **probability weighting function** $w(p)$ transforms objective probabilities p into **decision weights** $w(p)$. Its key characteristics are:

1. **Boundary conditions:** $w(0) = 0$ and $w(1) = 1$.
2. **Overweighting of small probabilities:** $w(p) > p$ for small p — rare events receive more weight than their objective probability warrants.
3. **Underweighting of large probabilities:** $w(p) < p$ for large p — highly likely events are psychologically discounted.
4. **Inverse S-shape:** The function lies above the 45° diagonal for small p and crosses below it for large p.
5. **Subcertainty:** $w(p) + w(1-p) < 1$ for most intermediate values of p.
6. **Certainty effect:** $w(1) = 1$ — certainty is treated as a special case and is not subject to probabilistic discounting.

The probability weighting function in Prospect Theory is typically specified as:

$$w(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{1/\delta}}$$

where $\delta > 0$ is a parameter that determines the curvature of the weighting function.

An alternative, simpler form commonly used is:

$$w(p) = p^{\delta}$$

where $\delta \in (0, 1]$ controls the degree of probability distortion.

Linear Probability as a Special Case

Linear probability weighting is the **special case when** $\boxed{\delta = 1}$:

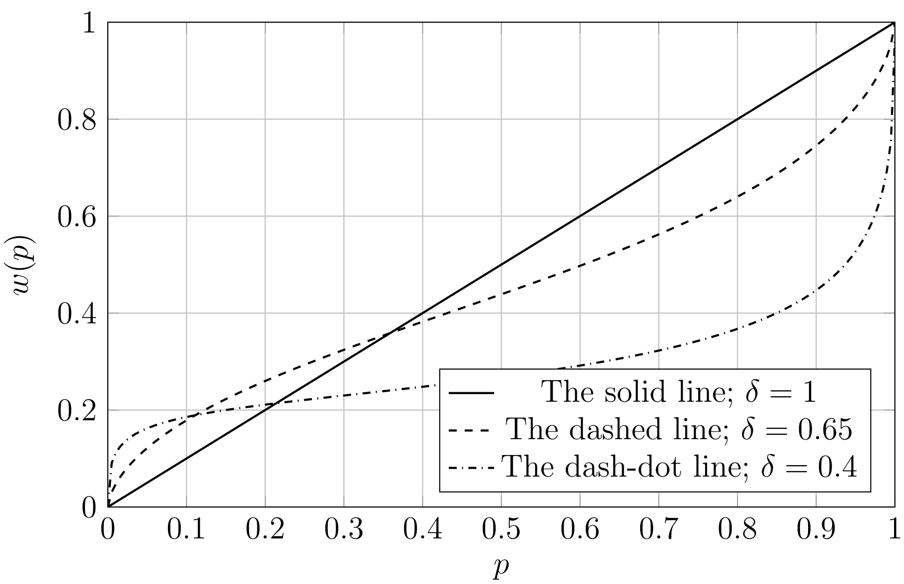
$$w(p) = p^1 = p$$

In this special case:

- The weighting function is the **identity function**—probabilities are perceived exactly as they are.
- Decision-makers weight outcomes by their **objective probabilities** with no distortion.
- This corresponds to **Expected Utility Theory (EUT)**, the rational benchmark model.
- There is no overweighting of rare events or underweighting of likely events.

The parameter $\delta \in (0, 1]$ captures how distorted the perception of objective probabilities is. The closer δ is to 0, the more curved the probability weighting function.

δ Determines the Curvature of $w(p)$



(3) Implications for Risk Attitudes

The two functions together imply that risk attitudes are **not uniform**; they depend on whether outcomes are gains or losses and on whether probabilities are small or large:

- The **concave value function in gains** $\Rightarrow$ risk aversion for moderate-to-high probability gains.
- The **convex value function in losses** $\Rightarrow$ risk seeking for moderate-to-high probability losses.
- **Overweighting of small probabilities** $\Rightarrow$ risk seeking for low-probability gains (e.g., buying lottery tickets) and risk aversion for low-probability losses (e.g., buying insurance).

(4) The Four-Fold Pattern of Risk Attitudes

	Gains	**Losses**
High Probability	**Risk Averse.** Prefer certain gain over gamble. *"Fear of disappointment."* *Example:* Accept \$900 over a 95% chance of \$1,000.	**Risk Seeking.** Prefer gamble over certain loss. *"Hope to avoid loss."* *Example:* Prefer a 95% chance of losing \$1,000 over a certain loss of \$900.
Low Probability	**Risk Seeking.** Prefer gamble over certain small gain. *"Hope of large gain."* *Example:* Buy a lottery ticket.	**Risk Averse.** Prefer certain small loss over gamble. *"Fear of large loss."* *Example:* Buy insurance.

This four-fold pattern arises from the **interaction** of the S-shaped value function (producing the gain/loss asymmetry) and the inverse-S probability weighting function (producing the small/large probability asymmetry).

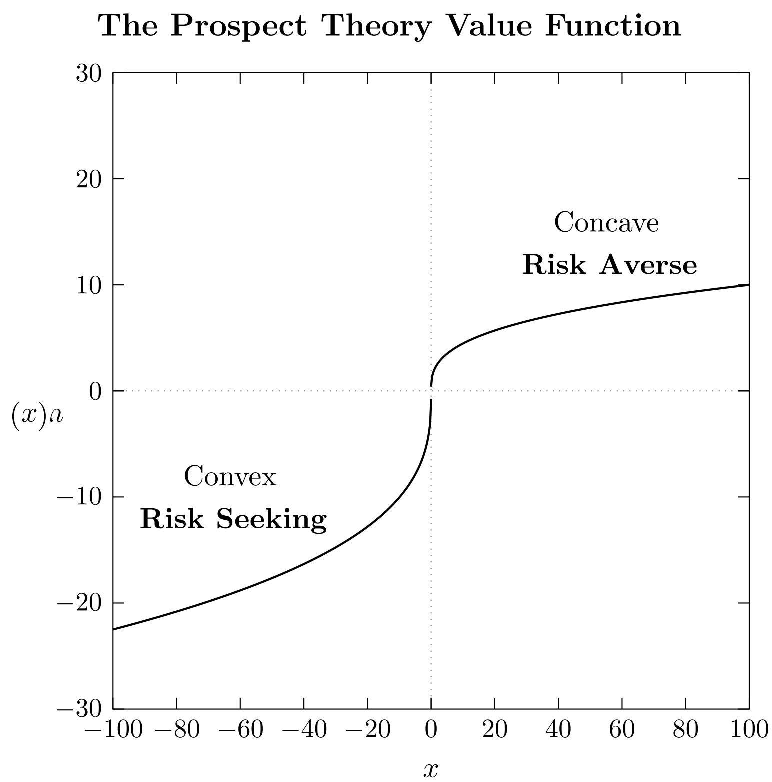


Figure 3: The Prospect Theory value function $v(x)$, plotted with exponent $\alpha = \beta = 0.35$ for visual clarity over the range $x \in [-100, 100]$. The function is strictly *concave* for gains ($x > 0$), implying *risk aversion*, and strictly *convex* for losses ($x < 0$), implying *risk seeking*. The loss side is everywhere steeper in absolute value, reflecting *loss aversion* ($\lambda \approx 2.25$).

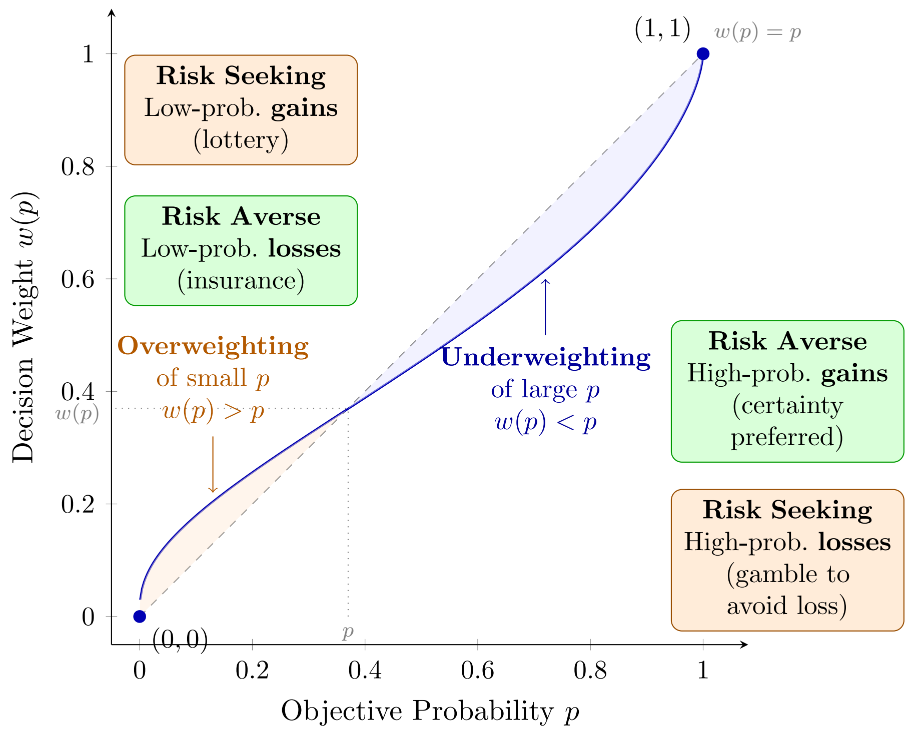


Figure 4: The Prospect Theory **Probability Weighting Function** $w(p)$, plotted using the Prelec (1998) one-parameter specification $w(p) = \exp\bigl(-(-\ln p)^{0.65}\bigr)$. The function is inverse S-shaped: it lies *above* the 45° diagonal for small probabilities (*overweighting*) and *below* it for large probabilities (*underweighting*), crossing at p^*. The four-fold pattern of risk attitudes is indicated in the shaded boxes, arising from the interaction of overweighting/underweighting with the gains/loss domain of the value function.

Question 12: Evolution of Models of Choice Under Risk

Guiding Question

Q12. Summarize how models of choice under risk have evolved from the Expected Value (EV) decision rule to Expected Utility Theory (EUT), and finally to Prospect Theory. In your answer, identify the key behavioral anomalies or empirical puzzles that challenged each earlier model, and explain how the subsequent theory addressed those limitations.

Stage 1: Expected Value (EV)

The Model. The earliest decision rule holds that a rational agent should choose the option with the highest expected monetary value:

$$EV = \sum_i p_i \cdot x_i$$

The Challenge — St. Petersburg Paradox (Bernoulli, 1738). The St. Petersburg game has infinite EV, yet no one would pay more than a modest sum to play it. EV predicts unbounded willingness to pay, which is clearly wrong. This showed that people do not simply maximise expected monetary payoffs.

Stage 2: Expected Utility Theory (EUT)

The Model. Proposed by Bernoulli and axiomatised by von Neumann and Morgenstern (1944), EUT holds that agents maximise the **expected utility** of outcomes, where utility is a concave function of wealth:

$$EU = \sum_i p_i \cdot u(x_i)$$

The concavity of $u(\cdot)$ captures risk aversion and resolves the St. Petersburg Paradox, since $\mathbb{E}[u(x)]$ is finite even when $\mathbb{E}[x]$ is infinite.

Axiomatic Foundation. EUT rests on axioms including completeness, transitivity, continuity, and the **independence axiom**: the ranking of two lotteries should not change when both are mixed with a third lottery in the same proportion.

The Challenges — Behavioural Anomalies.

1. **Allais Paradox (1953):** The common pattern of choosing B over A but C over D violates the independence axiom. People systematically overweight certainty.
2. **Framing Effect (Kahneman & Tversky, 1981):** Objectively identical choices yield different decisions depending on how options are framed. EUT requires description invariance.
3. **Reflection Effect:** Risk preferences systematically reverse between gains and losses. EUT with a globally concave $u(\cdot)$ cannot reproduce this.

4. **Reference Dependence:** Choices depend on the current reference point (status quo), not just final wealth levels.
5. **Isolation Effect:** People focus on the differentiating features of options and ignore shared components, violating the consistent probability evaluation required by EUT.

Stage 3: Prospect Theory (PT)

The Model. Developed by Kahneman and Tversky (1979) and extended to Cumulative Prospect Theory (1992), PT replaces EUT with two key innovations.

1. **The Value Function** replaces the utility function:
   - Defined over **gains and losses** relative to a reference point.
   - **S-shaped**: concave for gains, convex for losses.
   - **Loss averse**: losses loom larger than equivalent gains.
2. **The Probability Weighting Function** replaces objective probabilities:
   - Small probabilities are **overweighted**.
   - Large probabilities are **underweighted**.
   - Captures the certainty effect and the possibility effect.

How PT Addresses Each Anomaly:

Anomaly	**How PT Resolves It**
St. Petersburg Paradox	Concave value function for gains yields a finite valuation of the game.
Allais Paradox	Probability weighting overweights certainty ($w(1) = 1$), explaining the preference for B.
Framing Effect	Value function is defined over gains/losses; changing the frame changes whether outcomes are perceived as gains or losses, altering choices.
Reflection Effect	Concave in gains $\Rightarrow$ risk aversion; convex in losses $\Rightarrow$ risk seeking. The reversal is a direct prediction of the S-shaped value function.
Reference Dependence	The reference point is explicitly built into the model as the origin of the value function.
Isolation Effect	PT models how people simplify problems by focusing on distinctive elements, consistent with how Stage 1 is mentally discarded in the sequential framing.

Summary

The evolution of models of choice under risk can be summarised as follows:

Model	**Key Innovation**	**Key Limitation**
Expected Value (EV)	Maximise $\sum p_i x_i$	St. Petersburg Paradox: infinite EV yet people refuse to pay large sums to play.
Expected Utility Theory (EUT)	Maximise $\sum p_i u(x_i)$; concave $u(\cdot)$ captures risk aversion	Allais Paradox, Framing Effect, Reflection Effect, Reference Dependence, Isolation Effect.
Prospect Theory (PT)	S-shaped $v(\cdot)$ over gains/losses; probability weighting function $w(p)$	Still an active area of refinement; limited guidance under ambiguity.

Table 2: Evolution of models of choice under risk: key innovations and limitations.

Prospect Theory represents a major milestone in behavioural economics: it retained the mathematical rigour of EUT while incorporating realistic psychological features of human judgement, making it the most empirically successful model of decision-making under risk to date.