Week 10: Standard Intertemporal Choice

1 Overview

Intertemporal choice studies how individuals allocate resources across time. Individuals often face a choice between consuming today and consuming later. Consuming more today usually means consuming less in the future, while saving today allows for greater future consumption. Many important economic decisions involve trade-offs between present and future outcomes, such as saving, borrowing, education, investment, and retirement planning.

The standard model assumes that individuals are rational, forward-looking, and have stable preferences over time.

2 Present and Future Values of Cash Flows

2.1 Interest Rates

The interest rate measures the return to saving and the cost of borrowing.

- A higher interest rate increases the reward for saving.
- A higher interest rate also increases the cost of borrowing.
- The interest rate determines how present and future money are converted into one another.

2.2 Future Value

Future value measures how much an amount today will be worth in the future if it earns interest.

$$FV = PV(1 + r)^t$$

where:

- FV is future value,
- PV is present value,
- r is the interest rate,
- t is the number of periods.

2.3 Present Value

Present value measures how much a future amount is worth today.

$$PV = \frac{FV}{(1+r)^t} = (\frac{1}{1+r})^t FV = D(t)FV$$

The higher the interest rate, the lower the present value of a future payment.

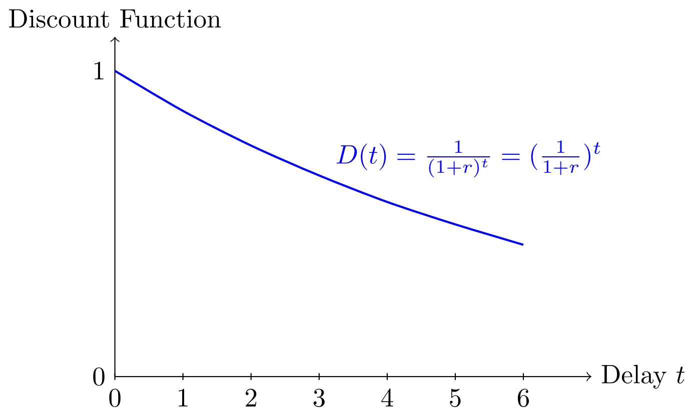

Figure 1: Financial discount function as a decision weight on future value

2.4 Net Present Value

Net present value is used to evaluate investment projects. It discounts all future costs and benefits back to the present.

$$NPV = \sum_{t=0}^{T} \frac{C_t}{(1+r)^t}$$

where C_t is the net cash flow in period t.

2.5 NPV Decision Rule

- If $NPV > 0$, accept the investment.
- If $NPV < 0$, reject the investment.
- If $NPV = 0$, the individual is indifferent.

Example: Investment Decision

Suppose an investment costs \$100 today and pays \$115 next year. The interest rate is 10%.

$$NPV = -100 + \frac{115}{1.10}$$

$$NPV = -100 + 104.55 = 4.55$$

Since $NPV > 0$, the investment should be accepted.

3 Discounted Utility: A Two-Period Model

Individuals choose consumption today (c_0) and consumption tomorrow (c_1) to maximize their discounted utility.

3.1 Intertemporal Budget Constraint

In a two-period model, the consumer faces the budget constraint:

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r}$$

where:

- c_0 and c_1 are consumption today and tomorrow,
- y_0 and y_1 are income today and tomorrow,
- r is the market interest rate.

Figure 2 illustrates the intertemporal budget constraint. The individual chooses a point on the budget line. Moving down the line means consuming more today and less in the future. Moving up the line means saving more today and consuming more in the future.

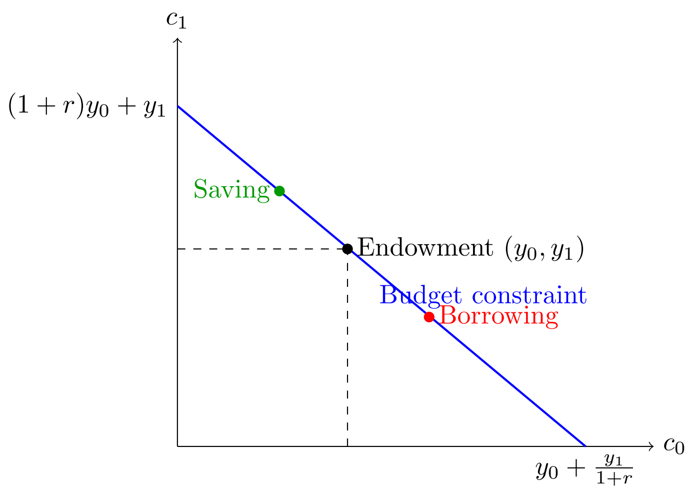

Figure 2: The intertemporal budget constraint

3.2 Borrowing and Saving

The budget constraint can also be written as:

$$c_1 = (1+r)(y_0 - c_0) + y_1$$

- If $c_0 < y_0$, the individual saves.
- If $c_0 > y_0$, the individual borrows.
- If $c_0 = y_0$, the individual neither borrows nor saves.

3.3 Optimal Consumption Choice

The individual chooses c_0 and c_1 to maximize:

$$U = u(c_0) + \delta u(c_1)$$

subject to:

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r}$$

At the optimum:

$$\frac{u'(c_0)}{\delta u'(c_1)} = 1 + r$$

This condition means that the marginal rate of substitution between present and future consumption equals the market trade-off determined by the interest rate.

3.3.1 Derivation of the Optimal Consumption Condition (Optional)

The individual chooses present consumption c_0 and future consumption c_1 to maximize lifetime utility:

$$U = u(c_0) + \delta u(c_1)$$

subject to the intertemporal budget constraint:

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r}$$

where r is the market interest rate and δ is the subjective discount factor.

Step 1: Write the Lagrangian

We can solve the constrained optimization problem using the Lagrangian method.

$$\mathcal{L} = u(c_0) + \delta u(c_1) + \lambda \left(y_0 + \frac{y_1}{1+r} - c_0 - \frac{c_1}{1+r} \right)$$

Here, λ is the Lagrange multiplier. It measures the marginal value of relaxing the lifetime budget constraint.

Step 2: Take First-Order Conditions

Differentiate the Lagrangian with respect to c_0, c_1, and λ.

First, with respect to c_0:

$$\frac{\partial \mathcal{L}}{\partial c_0} = u'(c_0) - \lambda = 0$$

Therefore:

$$u'(c_0) = \lambda$$

Second, with respect to c_1:

$$\frac{\partial \mathcal{L}}{\partial c_1} = \delta u'(c_1) - \frac{\lambda}{1+r} = 0$$

Therefore:

$$\delta u'(c_1) = \frac{\lambda}{1+r}$$

Third, with respect to λ:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = y_0 + \frac{y_1}{1+r} - c_0 - \frac{c_1}{1+r} = 0$$

This gives back the budget constraint:

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r}$$

Step 3: Eliminate the Lagrange Multiplier

From the first first-order condition:

$$\lambda = u'(c_0)$$

From the second first-order condition:

$$\delta u'(c_1) = \frac{\lambda}{1+r}$$

Multiply both sides by $(1+r)$:

$$(1+r)\delta u'(c_1) = \lambda$$

Since $\lambda = u'(c_0)$, we get:

$$u'(c_0) = (1+r)\delta u'(c_1)$$

Rearranging:

$$\frac{u'(c_0)}{\delta u'(c_1)} = 1+r$$

This is the optimality condition.

Economic Interpretation

The left-hand side is the marginal rate of substitution between present and future consumption:

$$MRS_{0,1} = \frac{u'(c_0)}{\delta u'(c_1)}$$

The right-hand side is the market trade-off between present and future consumption:

$$1+r$$

Therefore, at the optimum:

$$MRS_{0,1} = 1+r$$

This means the consumer chooses consumption so that the personal willingness to trade future consumption for present consumption equals the market rate of exchange.

Intuition

Suppose the individual gives up one unit of consumption today. If saved at interest rate r, it becomes:

$$1 + r$$

units of consumption tomorrow.
Therefore, the individual compares:

- the marginal utility lost from consuming one less unit today,
- with the discounted marginal utility gained from consuming $1 + r$ more units tomorrow.

At the optimum, these must be equal:

$$u'(c_0) = \delta(1 + r)u'(c_1)$$

If this condition does not hold, the individual can improve utility by changing consumption across time.

What Happens Away from the Optimum?

If:

$$u'(c_0) > \delta(1 + r)u'(c_1)$$

then the marginal utility of present consumption is high relative to future consumption. The individual should consume more today and save less.
If:

$$u'(c_0) < \delta(1 + r)u'(c_1)$$

then the discounted marginal benefit of saving is high. The individual should consume less today and save more.
At the optimum:

$$u'(c_0) = \delta(1 + r)u'(c_1)$$

3.3.2 The Two-Period Tradeoff: Impatience versus Consumption Smoothing

Consider a consumer who chooses consumption in two periods, c_0 and c_1, to maximize lifetime utility:

$$U = u(c_0) + \delta u(c_1),$$

where $0 < \delta < 1$. The parameter δ captures impatience: because future utility is multiplied by δ, one unit of utility tomorrow is worth less than one unit of utility today.
This model contains two opposing forces.

Impatience

Since $\delta < 1$, the consumer naturally prefers utility today over utility tomorrow. If utility were linear, this impatience would push the consumer toward consuming everything in period 0. Period 1 consumption receives only δ times as much weight as period 0 consumption.
For example, if

$$\delta = 0.75,$$

then one unit of utility tomorrow is worth only 0.75 units of utility today. Equivalently, it would take

$$\frac{1}{0.75} = 1.33$$

units of future utility to compensate for giving up one unit of present utility.

Diminishing Marginal Utility

However, consumption usually has diminishing marginal utility:

$$u'(c) > 0, \qquad u''(c) < 0.$$

This means that the first units of consumption are very valuable, but additional units become less valuable. For example, eating one meal when hungry gives a lot of utility, but eating a fifth meal on the same day gives much less additional utility.

Therefore, even though the consumer is impatient, it may not be optimal to consume everything today. If c_0 is very high and c_1 is very low, then the marginal utility of period 0 consumption is low, while the marginal utility of period 1 consumption is high. At some point, it becomes worthwhile to defer some consumption to the future.

This is the force behind consumption smoothing. The consumer balances the desire to consume sooner against the benefit of spreading consumption across periods.

3.3.3 Why Indifference Curves Are Convex

The tradeoff between impatience and diminishing marginal utility leads to convex indifference curves over intertemporal consumption bundles.

An indifference curve shows combinations of c_0 and c_1 that give the same lifetime utility:

$$U = u(c_0) + \delta u(c_1).$$

Along an indifference curve, if the consumer has a lot of present consumption and very little future consumption, then $u'(c_0)$ is low and $u'(c_1)$ is high. The consumer is willing to give up a large amount of present consumption to gain a small amount of future consumption.

By contrast, if the consumer has little present consumption and a lot of future consumption, then $u'(c_0)$ is high and $u'(c_1)$ is low. The consumer requires a large increase in future consumption to give up a small amount of present consumption.

This changing willingness to substitute consumption across periods makes the indifference curves convex. Intuitively, the consumer prefers a balanced consumption path to an extreme path. For example, the bundle

$$(c_0, c_1) = (50, 50)$$

is usually preferred to

$$(c_0, c_1) = (100, 0),$$

even if total consumption is the same, because the marginal utility of consumption is higher in the period with low consumption.

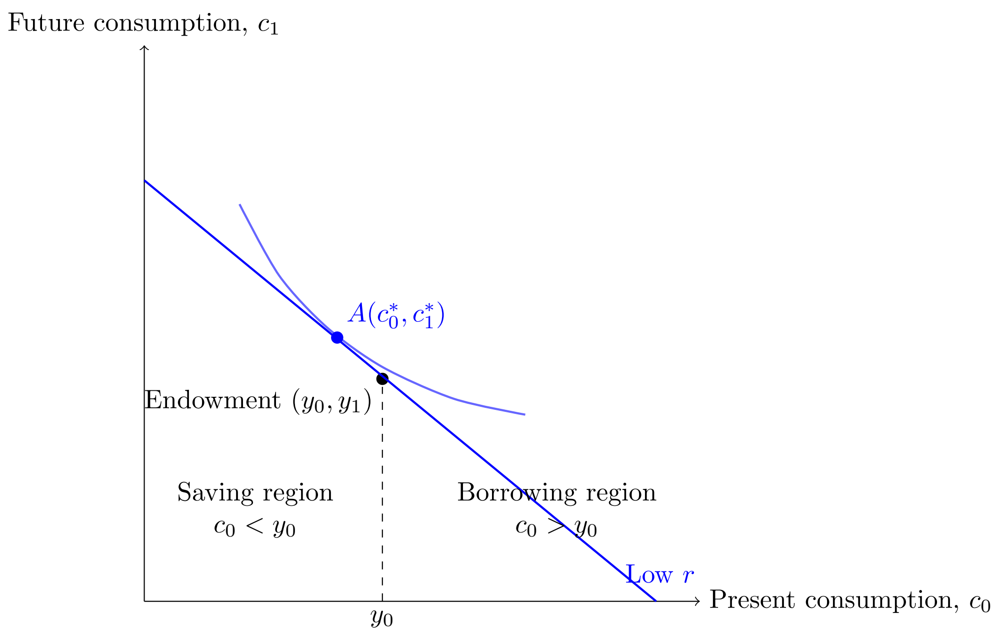

Figure 3: Optimal Consumption Plan

4 The Discounted Utility Model

4.1 Utility of Consumption Over Time

In the standard model of intertemporal choice, individuals choose consumption in each period to maximize total discounted utility.

$$U = \sum_{t=0}^{T} \delta^t u(c_t)$$

where:

- c_t is consumption in period t,
- $u(c_t)$ is the instantaneous utility from consumption in period t,
- $\delta \in (0, 1)$ is the discount factor.

4.2 The Discount Factor

The **discount factor** δ lies between 0 and 1: The discount factor captures how much future utility is valued relative to present utility. If δ is close to 1, the individual cares almost as much about future utility as present utility. If δ is close to 0, the individual puts very little weight on future utility.

$$\delta = \frac{1}{1 + \rho}$$

where ρ is the **subjective discount rate**, which is assumed to be non-negative ($\rho \geq 0$).

- A high δ means the individual is patient.
- A low δ means the individual is impatient.

Question: Why a high δ means the individual is patient, and a low δ means the individual is impatient?

Numerical Example: Patient and Impatient Individuals

To see how the discount factor reflects patience, suppose an individual compares receiving utility of 100 today with receiving utility of 100 one year from now.

The discounted value of future utility is:

$$\delta u(c_1)$$

Suppose the future utility is:

$$u(c_1) = 100$$

Now compare two individuals.

Patient Individual

A patient individual has a high discount factor. Suppose:

$$\delta = 0.95$$

Then the discounted value of future utility is:

$$0.95 \times 100 = 95$$

This person values future utility almost as much as present utility.

Impatient Individual

An impatient individual has a low discount factor. Suppose:

$$\delta = 0.50$$

Then the discounted value of future utility is:

$$0.50 \times 100 = 50$$

This person places much less weight on future utility.

Therefore, a high δ means the individual is relatively patient, while a low δ means the individual is relatively impatient.

4.3 Exponential Discounting

The standard model assumes exponential discounting:

$$U = u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \cdots + \delta^T u(c_T)$$

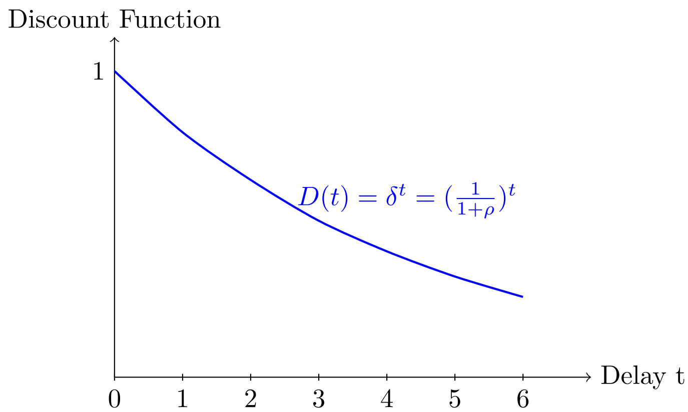

Figure 4: Exponential discounting with a constant discount rate

4.4 Properties of the Discount Function under Exponential Discounting

A discount function $D(t)$ gives the decision weight placed on an outcome that occurs t periods in the future. In the standard discounted utility model, outcomes that occur immediately receive full weight, while outcomes further in the future receive less weight.

The most common discount function used in economics is the exponential discount function:

$$D(t) = \delta^t,$$

where

$$0 < \delta \leq 1.$$

Here, δ is the subjective discount factor. If δ is close to 1, the individual is patient and places high weight on future utility. If δ is close to 0, the individual is impatient and places little weight on future utility.

The exponential discount function has several important properties.

Full Weight on the Present

The present receives full weight:

$$D(0) = \delta^0 = 1.$$

This means that utility received immediately is not discounted. For example, if a person receives utility of 100 today, its discounted value today is still 100.

Future Outcomes Receive Less Weight

For any future period $t > 0$,

$$D(t) = \delta^t < 1.$$

Thus, future utility is worth less than present utility. For example, if $\delta = 0.90$, then

$$D(1) = 0.90, \qquad D(2) = 0.81, \qquad D(3) = 0.729.$$

So utility received three periods from now receives only 72.9 percent of the weight of utility received today.

The Discount Function Is Decreasing

Because $0 < \delta < 1$, the discount function decreases with delay:

$$D(t+1) < D(t).$$

The further away an outcome is, the less weight it receives. Intuitively, a reward tomorrow is valued less than the same reward today, and a reward ten years from now is valued less than the same reward next year.

The Discount Function Approaches Zero

As the delay becomes very large, the discount function approaches zero:

$$\lim_{t\to\infty} D(t) = \lim_{t\to\infty} \delta^t = 0.$$

This means that outcomes in the very distant future receive very little weight today.

Constant Per-Period Discounting

A distinctive property of exponential discounting is constant per-period discounting. The ratio of discount weights between two adjacent periods is always the same:

$$\frac{D(t+1)}{D(t)} = \frac{\delta^{t+1}}{\delta^t} = \delta.$$

Thus, the proportional reduction in value from one period to the next is constant.

For example, if $\delta = 0.90$, then the relative weight of period 1 compared to period 0 is

$$\frac{D(1)}{D(0)} = 0.90.$$

The relative weight of period 6 compared to period 5 is also

$$\frac{D(6)}{D(5)} = 0.90.$$

So an exponential discounter treats "today versus tomorrow" in the same proportional way as "five years from now versus six years from now." This even-handedness across time is what generates time-consistent preferences.

4.5 Implications of the Standard Model (Exponential Discounting)

The standard discounted utility model has several important implications.

4.5.1 Time Consistency

Exponential discounting implies that the relative discounting between any two adjacent periods is constant.

$$\frac{\delta^{t+1}}{\delta^t} = \delta$$

Exponential discounting implies time consistency. A plan that is optimal today remains optimal in the future, as long as no new information arrives.

Suppose a person is choosing between receiving \$100 in 5 years and \$125 in 6 years. Today, the person prefers to wait for \$125 if

$$\delta^5 u(100) < \delta^6 u(125).$$

Dividing both sides by δ^5, this becomes

$$u(100) < \delta u(125).$$

Now suppose 5 years pass. The person is now choosing between \$100 today and \$125 one year from now. The person prefers to wait if

$$u(100) < \delta u(125).$$

This is exactly the same condition as before. Therefore, the person's preference does not reverse simply because time has passed.

A Numerical Example: Constant Relative Discounting

Suppose:

$$\delta = 0.90$$

The discount weights are:

$$D(0) = 1$$

$$D(1) = \delta = 0.90$$

$$D(2) = \delta^2 = 0.90^2 = 0.81$$

$$D(3) = \delta^3 = 0.90^3 = 0.729$$

Now compare the relative discounting between adjacent periods:

$$\frac{D(1)}{D(0)} = \frac{0.90}{1} = 0.90$$

$$\frac{D(2)}{D(1)} = \frac{0.81}{0.90} = 0.90$$

$$\frac{D(3)}{D(2)} = \frac{0.729}{0.81} = 0.90$$

The ratio is always:

$$\frac{D(t+1)}{D(t)} = \delta = 0.90$$

Thus, exponential discounting applies the same proportional reduction in value from one period to the next. This is why it implies a constant discount rate and time-consistent preferences.

A key implication of exponential discounting is that the relative discounting between any two adjacent periods is constant. When this is the case, a person's time preference is **time-consistent**. **Time consistency** means that your preference between two intertemporal options should not reverse with time. If preferences are time-consistent, a plan made today remains optimal in the future, as long as no new information arrives. If a person prefers option A over option B today for a future date, they should still prefer A over B when that future date arrives, as long as no new information appears.

For example, suppose today you prefer:

$$\$110 \text{ in 31 days}$$

over:

$$\$100 \text{ in 30 days}$$

Then, under exponential discounting, 30 days later you should prefer:

$$\$110 \text{ tomorrow}$$

over:

$$\$100 \text{ today}$$

The relative delay is the same, so preferences should not reverse.

4.5.2 Consumption Smoothing

Another implication is consumption smoothing. If individuals have concave utility functions, they prefer stable consumption over time rather than extreme fluctuations.

For example, most people would prefer:

$$(c_0, c_1) = (50, 50)$$

over:

$$(c_0, c_1) = (0, 100)$$

even though total consumption is the same.

4.5.3 Sensitivity to Interest Rates

The interest rate affects the price of consuming today versus consuming later. A higher interest rate makes saving more attractive because each dollar saved grows more in the future.

However, the effect of a higher interest rate on saving can be ambiguous because of two forces:

- **Substitution effect**: Higher interest rates encourage saving.
- **Income effect**: Higher interest rates make savers wealthier, which may increase present consumption.

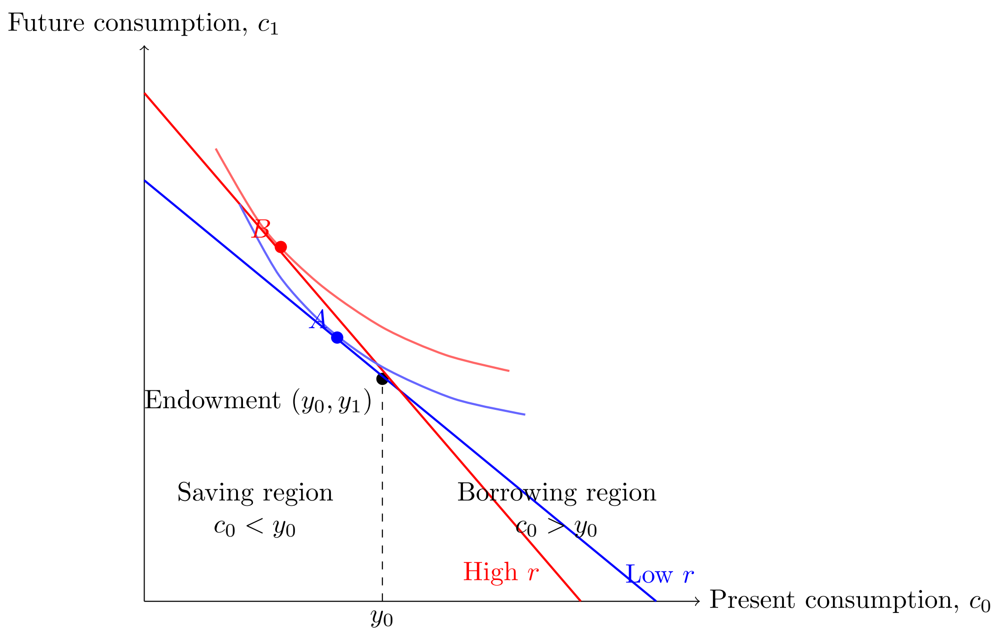

Figure 5: Effect of a higher interest rate on saving: substitution and income effects

4.6 Further Assumptions of the Standard Intertemporal Choice Model

The standard intertemporal choice model is built on several important assumptions. These assumptions make the model tractable and generate clear predictions, but they are also strong assumptions. Understanding them helps us see both the power and the limitations of the standard model.

4.6.1 Integration

Integration means that individuals evaluate new choices by integrating them with their existing consumption plans. A new opportunity is not evaluated in isolation. Instead, the individual asks how it changes consumption across all periods.

For example, suppose a person is offered the following investment:

Pay \$5,000 today and receive \$10,000 in five years.

Integration means the person does not simply ask, "Is \$10,000 larger than \$5,000?" Instead, the person considers how paying \$5,000 today affects current consumption, future consumption, borrowing, saving, and lifetime wealth.

If the person already has high current consumption and low future consumption, this investment may be attractive because it moves consumption toward the future. But if the person is currently liquidity constrained and needs money today, the same investment may be unattractive.

Thus, integration means that decisions are evaluated as part of the entire lifetime budget problem.

4.6.2 Utility Independence

Utility independence means that the decision maker cares only about the discounted sum of per-period utilities:

$$U = u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \cdots + \delta^T u(c_T).$$

The person does not have a separate preference over the pattern of utilities across time. All that matters is the total discounted utility.

For example, suppose two consumption plans generate the same discounted utility, but one gives a smooth emotional path while the other gives a "roller coaster" path of very high utility in one period and low utility in another. Under utility independence, the person is indifferent between them if the discounted sum is the same.

This assumption rules out preferences such as:

- preferring utility to increase over time,
- preferring a smooth utility profile,
- preferring to avoid emotional ups and downs,
- preferring to save the best experience for last.

For instance, many people prefer a vacation with the best day at the end rather than at the beginning. Utility independence rules out this kind of preference unless it affects the per-period utility numbers themselves.

4.6.3 Consumption Independence

Consumption independence means that utility from consumption in one period depends only on consumption in that period. Consumption in other periods does not directly affect today's instantaneous utility.

Formally, the standard model assumes:

$$U = u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \cdots + \delta^T u(c_T),$$

where $u(c_t)$ depends only on c_t, not on c_{t-1} or c_{t+1}.

For example, suppose a person chooses between Mexican food and Vietnamese food tonight. Consumption independence means that tonight's utility depends only on what the person eats tonight. It is not affected by whether the person ate Mexican food last night or expects to eat Vietnamese food tomorrow.

This assumption rules out habits, addiction, satiation, and reference dependence. For example:

- If drinking coffee today increases the desire for coffee tomorrow, consumption independence fails.
- If eating pizza yesterday makes pizza less enjoyable today, consumption independence fails.
- If exercising regularly makes future exercise more enjoyable, consumption independence fails.
- If current happiness depends on whether consumption is higher or lower than past consumption, consumption independence fails.

The key idea is that consumption independence rules out direct links between consumption in different periods.

4.6.4 Stationary Intertemporal Choice

Stationary intertemporal choice means that the instantaneous utility function is stable over time. The same consumption activity generates the same utility whenever it occurs.

For example, if eating an apple gives utility $u(\text{apple})$ today, then it gives the same utility tomorrow, next week, and next year, unless the amount consumed changes. The person's tastes are assumed to be stable.

This is different from consumption independence. Consumption independence says that today's utility does not depend on what was consumed yesterday or what will be consumed tomorrow. Stationarity says that the utility function itself does not change over time.

For example, suppose a person likes coffee today but dislikes coffee next year because their tastes change. This violates stationarity. By contrast, suppose the person likes coffee less today because they already drank five cups this morning. That violates consumption independence.

Stationarity rules out predictable changes in tastes, such as:

- becoming more health-conscious with age,
- developing a taste for bitter foods over time,
- enjoying nightlife less as one gets older,
- valuing leisure more after having children,
- having different preferences when stressed, hungry, or tired.

Under stationary intertemporal choice, the same utility function applies in every period. Therefore, changes in choices over time are explained by changes in constraints, prices, income, or discounting, not by changes in tastes.

5 Why Do People Discount the Future?

There are several reasons why people may value future outcomes less than immediate outcomes.

1. **Impatience**

   People often prefer immediate pleasure. Receiving money, food, entertainment, or comfort now may feel more valuable than receiving the same benefit later.

2. **Uncertainty**

   The future is uncertain. A promised reward in the future may not arrive, or personal circumstances may change.

3. **Opportunity Cost**

   Money today can be invested and earn interest. Therefore, a dollar today is worth more than a dollar tomorrow.

4. **Psychological Distance**

   Future outcomes may feel abstract or emotionally distant. People may care less about their future selves than their present selves.

5. **Anticipation and Memory**

   Future consumption may also generate utility before it happens through anticipation. Past consumption can generate utility through memory. These psychological components are not always captured by the standard model.

6 Key Takeaways

- Intertemporal choice involves trade-offs between present and future consumption.
- Interest rates determine the price of consuming today versus tomorrow.
- Present value and future value allow comparison of money across time.
- NPV provides a rule for evaluating investment decisions.
- The standard discounted utility model uses exponential discounting.
- Exponential discounting implies time-consistent preferences.